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Capital Requirements and Rational Discount-Window Borrowing

When banks face capital regulations and stochastic deposit supply, their decisions to borrow at the discount window will be affected by a broader range of variables than previous theoretical and empirical studies have recognized. Moreover, those decisions can respond discontinuously to changes in market parameters and to the form of rationing rule by which the discount window is administered. Risk aversion can complicate these linkages considerably, even causing some banks to prefer a positive discount rate that may exceed the actual level.

RECENT YEARS HAVE WITNESSED a disruption of historical linkages between the level of discount window adjustment borrowing and spreads between the discount rate and the cost of alternative sources of funds (see Mitchell and Pearce 1992; Clouse 1994; Cosimano and Sheehan 1994). These developments suggest that a deeper understanding is needed of the factors influencing rational borrowing decisions by banks. Few prior studies have addressed this issue from first principles, though several have quantified the historical empirical patterns.[1]

Here we analyze rational adjustment (overnight) borrowing decisions by banks in a two-stage stochastic framework that incorporates regulatory capital requirements and rate-setting behavior on the deposit side. Shaffer (1997) demonstrated in a similar framework that capital requirements can have a fundamental influence on banks' profit-maximizing funding strategies, though the focus of that study was on the effects, rather than causes, of borrowing behavior. Four extensions allow us to characterize various determinants of rational borrowing: increasing the number of stochastic factors to allow for unanticipated changes in all interest rates, imposing a discount window rationing rule similar to the current administration of adjustment credit, al-

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1. Goodfriend (1983) presents the most widely accepted theoretical analysis of a bank's decision to borrow, a dynamic optimization model based on banks' expectations of future federal funds rates and a nonprice discount window rationing rule; Waller (1990) models the interaction between Federal Reserve discount officers and banks. Empirical support for Goodfriend's model is found by Dutkowsky (1993) and, for large banks, by Peristiani (1994). The regional dynamic borrowing function estimated by Cosimano and Sheehan (1994) finds little or no empirical support for Goodfriend's model, but this could be an artifact of bank heterogeneity.

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lowing arbitrary probability distributions of an additive deposit supply shock, and considering arbitrary degrees of risk aversion among banks.

The results indicate that banks' optimal borrowing decisions depend on a broader set of variables than recognized in prior theoretical and empirical studies. In addition, those decisions can respond discontinuously to changes in parameters and to the form of nonprice rationing rule by which the discount window is administered. For example, a ceiling on the frequency of borrowing can, under some conditions, drive banks completely away from the discount window unless the discount rate is set at a subsidy level. Finally, risk aversion imposes a linkage between deposit interest rates and banks' choice of funding patterns not present under risk neutrality, and implies conditions under which banks prefer a strictly positive discount rate that may exceed the actual rate.

In comparing the model here with the standard model (for example, Goodfriend 1983) and with recent borrowing patterns, it is necessary to recognize two regulatory changes that occurred during the 1980s. First, the Regulation Q ceiling on deposit interest rates was phased out in the mid-1980s, having constituted a binding constraint from the late 1960s until it was phased out. Some degree of rate competition among banks for deposits, previously prohibited, is now a basic feature of the industry, and is incorporated in the model here but not in the standard model. Second, quantitative capital ratio requirements were adopted beginning in 1982, with risk-based capital requirements developed several years later. Like deposit rate competition, capital requirements were irrelevant to models characterizing banks' behavior prior to the 1980s.

A naive calibration of the model based on historically plausible parameter values yields contrasting predictions for the two types of capital requirements. Banks facing a leverage ratio requirement would always choose to avail themselves of the discount window in low-deposit states, but a risk-based capital requirement can induce some banks to avoid the discount window altogether. Thus, the adoption in the mid-to-late 1980s of the risk-based capital requirements, in an environment permitting deposit rate competition, may be one factor underlying the recent disruption of historical borrowing patterns, via a mechanism not reflected in previous models.

1. THE MODEL WITH EXPANDED UNCERTAINTY

As in Shaffer (1997), we depict banks' decisions as a two-stage process. In the first stage, a bank chooses its level of capital which, in conjunction with regulatory capital ratio requirements, determines an overall asset capacity; at the same time, the bank commits to a particular level of loans.[2] In the second stage, the bank chooses a deposit

2. Furlong and Keeley (1989) and other studies have similarly construed financial capital as one of the bank's choice variables, noting that the assumption of fixed capital is not appropriate for many larger banking organizations with access to capital markets. Moreover, regulatory constraints on allowable capital-to-asset ratios require that capital and assets be separately adjustable by the bank. The market value of a bank's capital may change ex post to reflect profit outcomes, but a bank can at least choose dividend payout ratios to prevent financial capital from rising above its desired levels. In repeated play, adverse shocks to profitability could conceivably reduce ex post capital below desired levels in ways not reflected in this type of model.

interest rate to attract a quantity of deposits sufficient to fund optimal asset levels. The deposit supply function includes a stochastic component that is realized exogenously after interest rates and loan levels have been chosen; the bank may respond ex post to various deposit outcomes by making instantaneous adjustments in its securities portfolio or by borrowing at the discount window, as necessary to satisfy its balance-sheet constraint. The bank's choices made at the first stage, in conjunction with the deposit state subsequently realized, determine the feasible set of these balance-sheet adjustments. The model is solved by the standard technique of backward induction, and also permits repeated play.

The basic model incorporates two deposit states, high and low, differing by an additive constant that may take bank-specific but exogenous values. (Section 3 below relaxes the assumption of two states.) It is convenient to categorize the bank's responses to these states in terms of two possible *cases,* or state-contingent patterns of funding, each associated with some optimal level of capital and loans as derived below. In case 1, the bank plans to hold securities ex post if the high state occurs. In case 2, the bank plans to borrow at the discount window ex post if the low state occurs. The bank must select its preferred case ex ante on the basis of expected profitability (or, in section 4 below, on the basis of a utility function incorporating both expected profits and the variability of profits). Thus, the bank's choices of capital levels, loan levels, deposit rates, and case are state independent. Shaffer (1997) showed that, under the assumptions listed below, the two cases are mutually exclusive—an optimizing bank will always choose a corner solution and will not both hold securities and borrow from the discount window.

Banks are assumed to be price takers in loans, securities, financial capital, and discount loans.[3] Except in the analysis of risk-averse banks in section 4, each of these prices will be considered stochastic—a generalization of the model of Shaffer (1997)—and uncorrelated with the deposit state. A stochastic loan rate could reflect changes in the yields of floating-rate loans tied to exogenous market rates. Stochastic yield on securities reflects the fact that the yield may change between the time the bank makes its funding decisions and the realization of the demand shock. A stochastic price of financial capital reflects the difficulty that a bank typically has in measuring this price with precision.[4] A stochastic discount rate is more general than required for realism, and is incorporated primarily to emphasize the model's flexibility. Under the assumed sequence of actions, the results are not affected by the stochastic nature of these variables, but only by the stochastic deposit supply. Analysis is carried out at the level of the banking firm, permitting a high degree of generality of market structure, strategic interactions among banks, and the degree of product or service differentiation on the deposit side. Resource costs are subsumed into the deposit interest rate.[5]

3. Price-taking behavior has been found empirically on the asset side not only for the U.S. banking industry (Shaffer 1989, 1996) but also for the much more concentrated Canadian banking industry (Nathan and Neave 1989; Shaffer 1993). By contrast, non-price-taking behavior has been found on the deposit side (Hannan and Liang 1993).

4. See Friedman and Kuttner (1992) and Hardouvelis and Wizman (1992) for more analysis of the cost of financial capital.

5. See Shaffer (1997) for further discussion and defense of these assumptions. The treatment of resource costs is comparable to that in VanHoose (1985).

In each case and in each state, bank i is subject to a balance-sheet constraint, $L_i + S_i \leq K_i + D_i(d_i)(1 - \delta) + F_i$, where L_i is the dollar volume of loans outstanding, $S_i \geq 0$ is the bank's dollar amount of securities owned, K_i is the bank's dollar amount of equity capital, d_i is the interest rate the bank pays for deposits, $\delta \in [0, 1)$ is the fractional reserve requirement, and $F_i \geq 0$ is the dollar amount of the bank's discount window borrowing.[6] $D_i(d_i)$ is the state-dependent supply of total deposits, assumed to be a continuous, monotone increasing function—which in this context says that a bank is able to attract additional deposits by some combination of actions that may include both price and nonprice dimensions (recalling that the interest rate is specified to include resource costs). Although it is convenient to assume that $D_i(d_i)$ is differentiable, such an assumption is nowhere used below and is not necessary so long as $D_i(d_i)$ satisfies some set of conditions that suffice to ensure the existence of equilibrium.

The deposit supply function in the high state exceeds that in the low state by a positive, bank-specific amount u_i. The high state occurs with exogenous probability α, the low state with probability $(1 - \alpha)$. We assume that all banks face the same state (high or low) on a given date, to avoid the need to model an interbank funds market; the model therefore reflects banks' funding shocks remaining after any interbank funds market has cleared (that is, net of federal funds transactions).[7] The assumption of a vertical supply shift (additive uncertainty) means that the bank is unable to influence the size of its random deposit outflows by its choice of deposit interest rate, even though its overall level of deposits (taking high and low states together) may respond to that choice; this assumption seems appropriate as a first approximation. Allowing bank-specific deposit shocks means that the model is able to accommodate the realistic situation in which larger banks may be subject to larger absolute deposit shocks than smaller banks.[8]

The regulatory leverage constraint is $K_i \geq k(L_i + S_i)$ where $k \in (0, 1)$. The appendix considers an alternative risk-based capital constraint, $K_i \geq kL_i$. Though both forms of constraint may be imposed simultaneously, as under current U.S. policy, only one of these can be binding at any time except on a set of measure zero. The state-dependent profit function is $\pi_i(d_i) = rL_i + sS_i - e_iK_i - d_iD_i(d_i) - fF_i$, where r is the interest rate on loans, s is the yield on securities, e_i is the price of equity capital, and f is the discount rate (all exogenous and stochastic). We use Greek letters to denote the expected values of these interest rates: ρ, σ, ε_i, and ϕ, respectively. We abstract from fixed costs, which do not alter the profit-maximizing conditions.

6. The reserve requirement is modeled as a true constraint that the bank must satisfy at all times. If the bank instead has the option of violating the requirement in some states and paying an associated penalty, the bank's optimization decision would include an additional step. The model also assumes that reserve requirements apply equally to all banks; however, as the conditions under which banks would self-select into a borrowing equilibrium versus a nonborrowing one are shown to be independent of the reserve requirement, all the results of this paper would hold even if a separate reserve requirement were set for each bank, including a zero requirement for some and positive requirements for others.

7. Thus, the model focuses on flows between the banking sector and other sectors, rather than on interbank flows. This focus is consistent with the role of open market operations in monetary policy, which represents a flow between the central bank and the banking sector, and with the market for banks' equity, which must ultimately be funded from outside the banking sector. The factors affecting banks' optimal choice of cases and their associated use of the discount window are not qualitatively altered by this abstraction.

8. Tobin (1982) also analyzed uncertainty, but for different purposes and assuming multiplicative rather than additive shocks, which requires that a bank be able to influence the size of its shock through its choice of deposit interest rates.

The exogeneity of r, s, and e_i imposes a linear structure on associated components of the bank's objective function, implying that the optimizing bank will operate at a corner solution with respect to S_i and F_i; we depict this situation by considering two separate cases, as described below. We assume that discount officers administer access to the discount window in a way that precludes arbitrage, so that banks never borrow in the high state.[9] This policy requires a nonprice rationing rule whenever the discount rate is set at a subsidy level on average (in this model, whenever $\phi < \sigma$), although the form of that rule would be more sophisticated than a simple restriction on the frequency of borrowing as embodied in several recent studies. (Section 2 below explores the implications of a frequency-based rationing rule.) Although e_i is exogenous, it may vary across banks, reflecting investors' diverse informational costs and banks' unequal earnings volatility and (unmodeled) insolvency risk; it may also vary across the business cycle.

We solve the model separately for each case and characterize the pattern of rational borrowing by comparing cases. Bank subscripts are suppressed hereafter for brevity.

Case 1: Buying Securities in the High-Deposit State. In this case, the bank chooses a level of loans according to the level of deposits it receives in the low state. However, to avoid the opportunity cost of holding uninvested funds in the high state, the bank chooses a level of equity that permits it to invest its excess deposits u in securities ($S = u(1 - \delta)$) during the high state. The leverage constraint will therefore be binding only in the high state. Combining the balance-sheet and leverage constraints, we find

$$K = k[D_{low}(d) + u](1 - \delta)/(1 - k); \tag{1}$$

$$L = [D_{low}(d) + ku](1 - \delta)/(1 - k). \tag{2}$$

The associated profits in the high and low states, respectively, are

$$\pi(d) = D_{low}(d)[(r - ek)(1 - \delta)/(1 - k) - d] + u[s(1 - \delta - d)] + uk(r - e)(1 - \delta)/(1 - k) \tag{3}$$

$$\pi(d) = D_{low}(d)[(r - ek)(1 - \delta)/(1 - k) - d] + uk(r - e)(1 - \delta)/(1 - k) \tag{4}$$

with expected profit:

9. Regulation A of the Federal Reserve System restrict adjustment credit to be available from the discount window "only for appropriate purposes and after reasonable alternative sources of funds have been fully used" (12 CFR 201.3 (a)) and requires each Federal Reserve Bank to "keep itself informed . . . with a view to ascertaining whether undue use is being made of depository-institution credit for the speculative carrying of or trading in securities . . ." (12 CFR 201.6 (b)(1)), while other Federal Reserve documents define inappropriate use of the discount window as including borrowing "to take advantage of a differential between the discount rate and the rate of alternative sources of funds" and "to support a planned increase in or continued holdings of investments or loans" (Federal Reserve System 1994, p. 10). Thus, our assumption is consistent with official policy and with actual administration of the discount window.

$$E\pi(d) = D_{low}(d)[(\rho - \varepsilon k)(1 - \delta)/(1 - k) - d]$$
$$+ uk(\rho - \varepsilon)(1 - \delta)/(1 - k) + \alpha u[\sigma(1 - \delta) - d]. \quad (5)$$

Case 2: Borrowing from the Discount Window in the Low-Deposit State. In this case the bank chooses to hold no securities, lends according to the level of deposits received in the high state, and borrows in the low state an amount $F = u(1 - \delta)$ from the discount window as necessary to satisfy the balance-sheet constraint. It chooses a level of financial capital sufficient to satisfy the leverage constraint given the chosen level of lending. Combining the balance-sheet and leverage constraints, we find

$$K = k[D_{low}(d) + u](1 - \delta)/(1 - k); \quad (6)$$

$$L = [D_{low}(d) + u](1 - \delta)/(1 - k). \quad (7)$$

Profits are

$$\pi(d) = [D_{low}(d) + u][(r - ek)(1 - \delta)/(1 - k) - d] \quad (8)$$

$$\pi(d) = D_{low}(d)[(r - ek)(1 - \delta)/(1 - k) - d]$$
$$+ u[(r - ek)/(1 - k) - f](1 - \delta) \quad (9)$$

in the high and low states, respectively, yielding expected profits of

$$E\pi(d) = D_{low}(d)[(\rho - \varepsilon k)(1 - \delta)/(1 - k) - d]$$
$$+ u[(\rho - \varepsilon k)(1 - \delta)/(1 - k) - \alpha d - (1 - \alpha)\phi(1 - \delta)]. \quad (10)$$

The Bank's Choice of Cases

Table 1 depicts the bank's balance sheet in each case and each state. Once the bank has committed itself to a particular capitalization and balance-sheet structure, ex post realizations of r, s, e, and f cannot alter the bank's choices. Even if the outcomes cause the bank to prefer a different case, the binding balance-sheet and capital constraints along with the short-run fixity of loans and capital prevent the bank from changing cases. The optimal choice between cases 1 and 2 for a risk-neutral bank will depend on the expected values of r, s, e, and f.

Comparing equations (5) and (10), we see that expected profits differ across cases only in terms that do not involve d. This means that the bank's profit-maximizing choice of deposit rate is invariant across cases, a neutrality result that permits further comparisons to be drawn without particularizing the deposit supply function and solving explicitly for d. From equations (10) and (15), *the bank prefers case 2 to case 1 if and only if*

TABLE 1

THE BALANCE SHEET IN DIFFERENT CASES AND STATES

		High State	Low State
Case 1	Loans:	$[D_{low}(d)+ku](1-\delta)/(1-k)$	
	Securities:	$u(1-\delta)$	0
	Deposits:	$D_{low}(d)+u$	$D_{low}(d)$
	Borrowing	0	0
	Capital:	$k[D_{low}(d)+u](1-\delta)/(1-k)$	
Case 2	Loans:	$[D_{low}(d)+u](1-\delta)/(1-k)$	
	Securities:	0	0
	Deposits:	$D_{low}(d)+u$	$D_{low}(d)$
	Borrowing	0	$u(1-\delta)$
	Capital:	$k[D_{low}(d)+u](1-\delta)/(1-k)$	

$$\phi \leq (\rho - \alpha\sigma)/(1-\alpha). \tag{11}$$

This intuitive condition says that the bank will choose the borrowing equilibrium if the discount rate is low enough, and precisely quantifies "low enough." It can be rewritten as $\rho \geq \alpha\sigma + (1-\alpha)\phi$, which says that the bank will choose the borrowing equilibrium if the expected loan yield exceeds the expected cost of funding the incremental lending associated with the borrowing equilibrium [implied by equations (7) and (2)]. Here $\alpha\phi$ is an expected opportunity cost of not holding securities in the high state, and $(1-\alpha)\phi$ is an expected direct cost of borrowing in the low state. This condition is satisfied by typical historical values of the parameters, in which $\rho > \sigma$, $\rho > \phi$, and $0 < \alpha < 1$. If $\phi = \sigma$ (that is, not a subsidy rate), (11) reduces to $\sigma \leq \rho$, which is also typically satisfied. Thus, banks can prefer to borrow at the discount window even if the discount rate is not a subsidy rate.

The condition (11) indicates that rational borrowing decisions respond not only to the actual and expected spreads between f and the cost of alternative funding, as implied by the model of Goodfriend (1983) and incorporated in many recent empirical studies (such as Mitchell and Pearce 1992; Dutkowsky 1993; Peristiani 1994; Cosimano and Sheehan 1994), but also to the interest rate earned on loans and the probability of an adverse deposit supply shock. The equivalent condition derived in the appendix for risk-based capital requirements further includes the cost of financial capital and the regulatory capital requirement. This set of results suggests that recent studies have omitted potentially important explanatory variables.

Since plausible parameter values appear to satisfy condition (11), flipping between cases 2 and 1 might seem unlikely to explain the recently observed instability in the empirical borrowing function. However, this reasoning also would suggest that all banks would borrow with comparable frequency from the discount window—that is, would choose case 2—contrary to actual experience. One possible explanation might be that the value of f relevant to the bank's decision must include the sum of the discount rate and any nonpecuniary borrowing costs imposed by the Federal Reserve's administration of the discount window. If that sum is high enough to violate condition

(11) for some banks in some periods, then the model could in principle provide a rational explanation for the observed instability in borrowing behavior. For $f = 0.055$, nonpecuniary costs must reach 0.105 before banks would turn from borrowing to the securities equilibrium. While this figure appears high, occasional spikes in the fed funds rate have exceeded 0.24 in recent years, revealing that some banks prefer to pay such rates rather than borrow from the discount window. The appendix demonstrates that *risk-based* capital requirements may constitute a stronger deterrent to banks' borrowing for historically observed parameter values.

Note that condition (11) is independent of the yield on securities and therefore independent of the spread between f and s or between ϕ and σ. Moreover, like previous models of discount window borrowing, (11) is independent of δ.[10]

2. DISCOUNT WINDOW RATIONING

The previous section characterized the administration of the discount window as precluding arbitrage, without placing restrictions on the allowable frequency of access to the discount window per se. Footnote 10 documents that this interpretation is consistent with the written policy and regulations. However, other studies have described the actual practice of discount window administration as more akin to imposing a ceiling on the frequency of access (Goodfriend 1983; Meulendyke 1992). In this section, therefore, we explore how this alternative rationing rule would affect the bank's borrowing decision.

If the maximum allowable frequency of borrowing is comparable to the average frequency of low-deposit states, the rule would have a similar effect on banks as the no-arbitrage policy except when abnormally many consecutive low states occur. It is even plausible that a judiciously chosen frequency rule might be used in place of, rather than in addition to, a no-arbitrage rule to achieve the same outcome without requiring discount officers to monitor the deposit state, at least in a two-state world with known α; section 3 below briefly explores how a continuously distributed deposit shock might alter this conclusion.[11]

Here, we focus on the contrasting case in which banks are permitted to borrow with maximum frequency $\beta < (1 - \alpha)$. Then, in repeated play within the framework of the previous section, a bank would expect to face a low-deposit state more often than it could fund through the discount window. Its choice of capitalization and lending lev-

10. The independence results here are a positive finding, derived as the equilibrium outcome of a formal model, whereas in previous models they have comprised an assumption.

11. Waller (1990) assumes that discount officers derive disutility from deterring appropriate borrowing requests and from approving inappropriate requests, even if these opposing types of errors are made equally often and thus result in a frequency of actual borrowing that coincides with the frequency of appropriate needs to borrow. Although we do not explicitly model the utility of the discount officer, in this section we implicitly adopt the most optimistic assumption under which to assess rationing rules—namely, that only the frequency of borrowing must be matched to the frequency of legitimate funding needs, without regard to matching specific events period by period. The justification for this assumption is that, if a bank knows that inappropriate borrowing for arbitrage purposes today is likely to render it ineligible to borrow during a future period of legitimate need, its profit-maximizing decision will be to forego the inappropriate borrowing because of the higher cost of violating the reserve requirement later. If this assumption is not valid, then it becomes harder to justify a frequency-based rationing rule as optimal.

els must thus provide for alternative funding sources (case 1), allowing the bank to operate independently of the discount window.

As a result, whether it ever borrows depends on whether its profit in the low state exceeds that given by equation (4) above. As the bank's balance sheet would be configured for case 1 [that is, equations (1) and (2) above], its profit in the event of borrowing would be

$$\pi(d) = D_{low}(d)[(r - ek)(1 - \delta)/(1 - k) - d] + u(1 - \delta)[k(r - e)/(1 - k) + s - f]. \quad (12)$$

This expression exceeds the value given by equation (4) if and only if $u(s - f)(1 - \delta) > 0$, or $s > f$ (that is, if the discount rate is set at a subsidy rate). In this case, the bank will borrow up to the allowable fraction β of days.

Note that, because securities holdings and discount loans can both be adjusted instantaneously, the bank is able in this case to respond ex post to realized values of the various parameters in choosing between selling off its securities holdings in the low state versus borrowing at the discount window to sustain its asset portfolio. Therefore, there is some possibility that the bank may end up borrowing less often than β within any given number of days, depending on the realized outcomes of s and f. Also note that borrowing in this case would have the effect of sustaining the bank's securities holdings rather than loans; this outcome is consistent with current Federal Reserve policy and practice, as the bank is not actually expanding its holdings of securities or other assets at the time it borrows.

Considering both forms of capital requirements (see the appendix), we see that the principal finding of this section is that *if the discount window is administered with a binding, frequency-based rationing rule, banks will choose not to borrow at the discount window at all unless the discount rate is set at a subsidy level.* Here, the rationing rule is considered "binding" when it is more restrictive than a no-arbitrage condition, and a "subsidy level" is defined relative to yields on investment securities rather than relative to the federal funds rate. In practice, the intent of either definition of "subsidy"—to identify arbitrage opportunities in which the bank could borrow at a low rate from the discount window and simultaneously lend at a higher rate in a liquid market—would be the same.

3. GENERAL ADDITIVE DEPOSIT SHOCKS

In this section we relax the two-state assumption, positing a more general additive deposit shock of the form $D(d) = D_{low}(d) + z$ where z is a random variable with p.d.f. $g(z)$, $z \in [0, v]$. Then each bank can choose a threshold $\bar{z}$ separating different funding strategies, most of which correspond to the funding cases analyzed in section 1. In case 1 (buying securities), the bank can instantaneously adjust its securities holdings as needed, so this case goes through as in section 1 except that expected profit includes a term reflecting the expected or average deposit shock.

In case 2 (borrowing at the discount window), the bank can apportion its balance-sheet needs among the amount borrowed and the amount invested in securities. The possibilities here are more complex, as the bank can borrow from the discount window for all $z \in [0, \bar{z}]$ and invest in securities (which we call case 3) for $z \in [\bar{z}, v]$. In this case, the leverage and balance-sheet constraints imply

$$K = k[D_{low}(d) + v](1-\delta)/(1-k), \tag{13}$$

$$L = (1-\delta)[z + (D_{low}(d) + kv)/(1-k)], \tag{14}$$

$$E\pi = \rho(1-\delta)[\bar{z} + (D_{low}(d) + kv)/(1-k) - \varepsilon k(1-\delta)(D_{low}(d) + v)/(1-k) - dD_{low}(d) + \sigma(1-\delta)\int_{\bar{z}}^{v}(v-z)g(z)\,dz - d\int_{0}^{v} zg(z)\,dz - \phi(1-\delta)\int_{0}^{\bar{z}} zg(z)\,dz. \tag{15}$$

The first- and second-order conditions at the first stage, derived from equation (15), imply

$$\rho - \sigma v g(\bar{z}) + \bar{z}g(\bar{z})(\sigma - \phi) = 0 \tag{16}$$

$$-\sigma g'(\bar{z}) + g(\bar{z})(\sigma - \phi) + \bar{z}g'(\bar{z})(\sigma - \phi) < 0 \tag{17}$$

after dividing both sides of each condition by $(1 - \delta) > 0$. If the deposit shock is uniformly distributed, the second-order condition takes the sign of $g(\bar{z})(\sigma - \phi)$, implying a profit *minimum* for all $\phi < \sigma$ (that is, whenever the discount rate is expected to be a subsidy rate). Then the bank will choose the more profitable of the two corner solutions, $\bar{z} = 0$ (case 1) or $\bar{z} = v$ (case 2).

Overall, a variety of conditions can lead the bank to choose a corner solution even when a continuous distribution of deposit shocks would permit interior solutions in principle. Thus, the simple two-state version of the model in section 1 appears capable of reflecting a more common range of outcomes than might be suspected.

Administration of the discount window in this context can still take two forms. The above derivations assume the no-arbitrage restriction of section 1, suitably reinterpreted for the more general deposit shock. Alternatively, a ceiling on the frequency of borrowing can be represented as a ceiling on $\bar{z}$ exogenously given to the bank; then the bank would compare the expected profitability of its alternatives at this value versus at its preferred choice of $\bar{z}$, if that is lower than the ceiling, or versus $z = 0$ otherwise. As in the two-state model, the possibility arises that a binding administrative ceiling on the borrowing frequency (that is, $\bar{z}$ set below the bank's preferred value) could have the discontinuous effect that the bank chooses never to borrow at the discount window. Because not all banks will generally face the same distribution of deposit shocks, it is unlikely that a uniform administrative ceiling can be appropriate for all banks. Rather, any given administrative value of $\bar{z}$ is likely to be both too high to prevent occasional arbitrage opportunities for some banks and too low to be consistent with appropriate, rational use of the discount window by some other banks. More specific conclusions would require empirical study of the historical distribution of interday deposit shocks.

4. RISK-AVERSE BANKS

The analysis thus far has assumed that banks are risk neutral. We can extend this framework to risk-averse banks maximizing an objective function $U(\pi) = E\pi + \gamma\sigma_\pi^2$ where $\gamma < 0$ indexes the degree of risk aversion and σ_π^2 denotes the variance of profit. In the simplest case we assume that r, s, e, and f are nonstochastic and that there are two deposit states. Then $E\pi = \alpha\pi_H + (1-\alpha)\pi_L$ while $\sigma_\pi^2 = \alpha(1-\alpha)(\pi_H - \pi_L)^2$ so $U(\pi) = \alpha\pi_H + (1-\alpha)\pi_L + \gamma\alpha(1-\alpha)(\pi_H - \pi_L)^2$. For case 1, applying equations (3) and (4), this expression reduces to

$$U(\pi) = D_{low}(d)[(r-ek)(1-\delta)/(1-k) - d] + uk(r-e)$$
$$(1-\delta)/(1-k) + \alpha u(s - s\delta - d) + \gamma\alpha(1-\alpha)u^2(s - s\delta - d)^2. \quad (18)$$

For case 2, applying equations (8) and (9), the bank's objective function reduces to

$$U(\pi) = D_{low}(d)[(r-ek)(1-\delta)/(1-k) - d] + u[(r-ek)/$$
$$(1-k) - f](1-\delta) - \alpha u(d - f + f\delta) + \gamma\alpha(1-\alpha)u^2(d - f + f\delta)^2. \quad (19)$$

Interestingly, this expression suggests that a risk-averse bank would not want to see the discount rate set below some level, as the first-order condition $\partial U(\pi)/\partial f = 0$ indicates a maximum at $f = (1 + 2\gamma\alpha u d)/[2\gamma\alpha u(1-\delta)]$.[12] Larger values of f increase the bank's direct cost of borrowing; smaller values of f increase the variance of profits across states, an effect which—for a risk-averse bank—tends to offset the benefit of reduced borrowing costs.

Because these last two equations (as well as the corresponding equation in the appendix under risk-based capital requirements) differ from each other in terms involving d, the bank's preferred choice of d (and hence the quantity of deposits attracted) will differ in each case. Thus, risk aversion destroys the case invariance of the equilibrium deposit rate that characterizes risk-neutral banks in this framework. Consequently, we cannot identify in general which case or funding pattern risk-averse banks will choose, or even which cases would yield the higher deposit rates, apart from knowledge of the specific functional form of the deposit supply curve. However, for γ sufficiently close to zero (that is, for sufficiently small degrees of risk aversion), banks' funding preferences would coincide with the risk-neutral choices shown in section 1 above.

12. This value of f is positive for a sufficiently large deposit shock. For example, if $\alpha = 0.9$, $\gamma = -0.5$, and $d = 0.06$, then $f > 0$ for all $u > 18.52$. The second-order condition shows that the extremum is a maximum, since $\partial^2 U(\pi)/\partial f^2 = 2(1-\delta)^2\gamma\alpha(1-\alpha)u^2 < 0$ for all $\gamma < 0$.

For any given value of γ, the difference between the risk-averse and risk-neutral optimal pricing decisions is greatest for $\alpha = 1/2$ and declines as either $\alpha \rightarrow 0$ or $\alpha \rightarrow 1$. That is, the more time the economy spends in a particular deposit supply state, the less difference a bank's risk attitude makes regarding its rational pricing, funding, and borrowing decisions. For sufficiently large or small values of α, risk-averse banks' funding preferences would match the rational choices of risk-neutral banks shown in section 1. Little else can be usefully said about the effects of risk aversion in the absence of more specific information about the distribution of deposit shocks and the functional form of deposit supply.

5. CONCLUSION

Recent theoretical and empirical studies of discount window borrowing behavior have largely focussed on actual and expected spreads between the discount rate and the federal funds rate, and on the frequency of recent borrowing, as explanatory variables. The analysis of rational borrowing decisions in the presence of regulatory capital requirements indicates that a broader range of variables will influence those decisions, especially under risk-based capital requirements. Among the additional variables are the interest rate earned on loans, the cost of financial capital, the regulatory capital requirement, and the probability of an adverse deposit supply shock. Thus, existing studies (both theoretical and empirical) omit potentially important variables. The significance of this difference is emphasized in the appendix, where risk-based capital requirements (which did not exist when Goodfriend's 1983 model was developed) were shown to induce some banks to avoid the discount window in all deposit states for plausible parameter values.

Moreover, it was shown that borrowing decisions may respond discontinuously to a variety of conditions, even in some cases where all stochastic factors follow a continuous distribution. This finding suggests a further possible reason why it has proven difficult to forecast a smooth relationship between discount borrowing and various financial factors.

It was also shown that a nonprice rationing rule that is more restrictive than a no-arbitrage condition can cause banks to exhibit extreme reluctance to borrow at the discount window if the discount rate is not set at a subsidy level. This finding appears to have no relevance to current practice, but would have implications for any policy decision to combine a market-based discount rate with a ceiling on the frequency of access to the discount window.

Finally, the funding decisions of risk-averse banks were explored, but with relatively sparse conclusions. Risk aversion introduces a linkage between the bank's funding decision and its choice of deposit interest rate that does not exist for risk-neutral banks, and which implies that the specific deposit supply function and other parameter values must be known before meaningful comparisons can be drawn among alternative funding patterns. It was shown that risk-averse banks prefer a particular value for the discount rate that, for some parameter values, would be positive;

this characterization contrasts with the preferences of risk-neutral banks for unboundedly negative discount rates, and results from the trade-off between lower borrowing costs and a higher variance of profits.

APPENDIX: RISK-BASED CAPITAL REQUIREMENTS

If the regulatory capital constraint is risk-based, $K \geq kL$, then case 2 remains unchanged but the securities equilibrium (which, under a risk-based capital constraint, we call case 4) differs from case 1. The balance-sheet and risk-based capital constraints together imply

$$K = kD_{low}(d)(1 - \delta)/(1 - k); \tag{A1}$$

$$L = D_{low}(d)(1 - \delta)/(1 - k). \tag{A2}$$

Profits in the high and low states, respectively, are

$$\pi(d) = D_{low}(d)[(r - ek)(1 - \delta)/(1 - k) - d] + u[s(1 - \delta) - d]; \tag{A3}$$

$$\pi(d) = D_{low}(d)[(r - ek)(1 - \delta)/(1 - k) - d] \tag{A4}$$

yielding expected profits of

$$E\pi(d) = D_{low}(d)[(\rho - \varepsilon k)(1 - \delta)/(1 - k) - d] + \alpha u[\sigma(1 - \delta) - d]. \tag{A5}$$

Because equations (A5) and (10) differ only in terms that do not involve d, the bank's profit-maximizing choice of deposit rate is invariant across cases, just as under a simple leverage-based capital constraint. The following conclusions may then be drawn.

From equations (10) and (A5), *the bank prefers the borrowing equilibrium to the securities equilibrium if and only if*

$$\alpha\sigma + (1 - \alpha)\phi \leq (\rho - \varepsilon k)/(1 - k) \tag{A6}$$

in which the left-hand side is the expected cost of funding additional loans, as in section 1, while the right-hand side is a net yield on additional loans that must be capitalized. As in condition (11), condition (A6) indicates that a bank's decision to borrow from the discount window is a function of additional variables beyond the expected spread and frequency of recent borrowing. The right-hand side of (A6) differs from that of (11) by $k(\rho - \varepsilon)/[(1 - k)(1 - \alpha)]$ which has the sign of $\rho - \varepsilon$, typically negative. Thus, discount-window borrowing is preferred under a narrower range of discount rates when the capital requirement is risk based than when it is a simple leverage ratio.

In particular, condition (A6) can be violated by plausible parameter values, implying that some banks would want to avoid the discount window. In calibrating the model to demonstrate this result, several points must be noted. First, banks' buildup in recent years of capital levels often well in excess of the regulatory minima suggests that the perceived or effective value of k may exceed the nominal regulatory floor, and may vary by bank. One can suggest many reasons for this phenomenon, including banks' desire to maintain excess capital to fund acquisition opportunities or to satisfy the regulatory minima even after unanticipated losses, but the phenomenon is empirically well established regardless of its causes.

Second, if the price of equity capital is proxied by a bank's return on equity, then $\varepsilon = E\pi/K$, which in case 1 is given by the ratio of equations (A5)/(A1). Thus, $\varepsilon = \frac{1}{2}\{\rho/k - [d(1-k)(1+\alpha u/D_{low}(d))]/[k(1-\delta)] + \alpha u\sigma/[kD_{low}(d)]\}$. For example, if $\alpha = 0.9$, $k = 0.09$, $\rho = 0.07$ net of loan loss provisioning, $d = 0.04$, $\delta = 0.02$, $\sigma = 0.06$, and $u = 0.02D_{low}(d)$, we find $\varepsilon = 0.1794$, which is well within the range of performance exhibited by many individual banks in recent years. If $\phi = 0.055$, the left-hand side of (A6) is 0.0595 while the right-hand side is 0.0592, so the bank will choose case 1 and never borrow from the discount window. This model therefore suggests that the adoption of risk-based capital requirements in recent years might be one factor underlying the increased reluctance of banks to borrow.

If the rationing rule of section 2 is combined with a risk-based capital requirement, profits in "case 4 with borrowing" [corresponding to (12) under the alternative capital requirement] are

$$E\pi(d) = D_{low}(d)[(r - ek)(1-\delta)/(1-k) - d] + u(1-\delta)(s-f) \tag{A7}$$

which exceeds equation (A4) if and only if $s > f$, or the discount rate is a subsidy rate.

From equation (12), a risk-averse bank's objective function in case 4 is

$$U(\pi) = D_{low}(d)[(r - ek)(1-\delta)/(1-k) - d] + \alpha u[s(1-\delta) - d] + \gamma\alpha(1-\alpha)u^2[s(1-\sigma) - d]^2. \tag{A8}$$

As under a simple leverage requirement, this equation differs from equation (19) in terms involving d. Thus, again, the bank's choice of d (and associated quantity of deposits) will differ across cases.

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